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A. G. Broydo, N. A. Suboch'

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ESTIMATING THE ACCURACY OF A SIMPLE METHOD FOR CALCULATING THE HEAT BALANCE IN THE SOIL

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ABSTRACT. The different methods used to measure the heat held in the upper layers of the soil, or given up by those layers, over a particular time span are discussed and their accuracy evaluated.

1. General Considerations /94*

Data on the quantity of heat contained, or released, by the upper layers of the soil over a particular time span are needed to solve certain of the problems faced by the national economy. The instruments available for this purpose at this time are not simple enough in design to permit this to be done, so these data must be calculated. G. Kh. Tseytin [1] used the theory hypothecated by D. L. Laykhtman [2, 3][1] to develop one such method. Analysis of the theoretical bases of this method, as well as the comparison made by G. A. Voloshinova [4] between one of the first of its variants and certain of the methods of making the calculations proposed earlier, lead to the conclusion that the method described in [1] is, today, relatively more accurate. The method in [1] has been discussed by N. P. Rusin [5] in a somewhat simplified form, designed for use in a network.

Still, even in simplified form, G. Kh. Tseytin's method requires calculations that are, if not complex, at least time-consuming. The validity of the further simplification proposed by N. P. Rusin [6] evidently requires additional verification. So down to the present time there remains the very current problem of developing not only a sufficiently accurate, but also a simple, method of calculating fluxes of heat through the soil.

Here we are reminded of the work done by K. A. Sychev [7], who proposed an elementary method of calculating such fluxes that was based on the procedure

1. The authors were not aware of reference [10] at the time this article was written.

* Numbers in the margin indicate pagination in the foreign text.

used to determine the heat balance in the ocean, and developed by V. B. Shtokman. But it must be pointed out that the considerable specificity of the manner in which the heat exchange takes place between the surface and the deeper layers in the ocean, as compared to the corresponding processes in hard soil, means that special care must be exercised when the procedure used to make calculations designed for the ocean is "brought ashore." The results obtained for land using this procedure can be nothing other than approximate, as the author himself quite correctly points out. Specifically, the automatic replacement of the differentials by finite differences in the corresponding original equations can result in significant errors, given the vertical temperature distributions characteristic of soil. These differ greatly from linear, particularly near the surface, where the more intensive the heat exchange, the more intensive the change.

Thus it appears to be desirable to evaluate the possible errors in the method in [7]. The establishment of this evaluation also was the task of this paper, and the results obtained by the method in [1] were used as the criterion /95 for the comparison.

Soil temperature measurements made in three regions were used in the calculations:

(a) at the station for the physics of the ground layer of air in Koltushi, near Leningrad (July 1951);

(b) at the vapor field in Kamennaya Step' (July 1951) [8];

(c) in the semidesert in the vicinity of the Pakhta-Aral oasis in Central Asia (July 1952) [9].

Since we must know the coefficient of thermal conductivity in order to calculate fluxes of heat through the soil, it is desirable to begin with a consideration of the results of the calculations made for this coefficient in order to evaluate the error in the method in [7].

2. The Coefficient of Thermal Conductivity

Eq. (10) in reference [7] was used to calculate the coefficient of thermal conductivity by K. A. Sychev's method. Eq. (11) was not used for two reasons.

(a) According to the handling of the method in [1], as used in [2], the coefficient of thermal conductivity can be determined from soil temperature observations only during daylight hours, whereas K. A. Sychev's Eq. (11) is designed to determine the mean value of this magnitude over a time interval of the order of days, and longer, so could not be used to make the calculations, the results of which would be compared with those obtained from the method in [1].

(b) The physical sense of the magnitude obtained from this formula is not entirely evident because, as we know, the mean magnitude of the relationship cannot always be obtained by the simple averaging of the numerator and denominator separately, as evidently was the case in deriving Eq. (11).

The coefficients of thermal conductivity based on G. Kh. Tseytin's method, and calculated using the relationship flowing from Eqs. (2), (7), (8), and (8') in reference [1], have been used as the "control" values. Not used here is the somewhat simplified Eq. (9), recommended in [5], because of its lesser accuracy.

Determined in the course of processing all three groups of original data was the mean coefficient of thermal conductivity of the upper layer of the soil to a depth of 20 cm. Table 1 lists the results of the calculations and the comparison. The values obtained using K. A. Sychev's formula carry the designation a_S, those obtained using G. Kh. Tseytin's formula the designation a_{Ts}.

The results obtained for Koltushi for 4 and 19 July 1951, invite our attention. In the first case we have the greatest discrepancy in the values obtained using the two methods, whereas in the second case both methods result in negative values without physical meaning. One explanation of these results can be the sharply unstable nature of the weather on those days, during which there were alternating periods of sun and precipitation, resulting in sharp variations in the moisture of the upper soil layer, which, in turn, caused significant, and rapid, changes in the coefficient of thermal conductivity, which was unable to assume its "normal" magnitude.

Table 1 provides the following conclusions.

(a) K. A. Sychev's method reflects correctly the dynamics of the change in the coefficient of thermal conductivity in the majority of cases (in 20 out of

27, that is, in 74% of the cases).

(b) Generally speaking, the calculation made using K. A. Sychev's formula results in a lower coefficient of thermal conductivity than is the case when G. K. Tseytin's method is used to make the calculation (25 cases out of 29, or 86%).

(c) The absolute mean deviation in all calculated a_S values from the corresponding a_{Ts} values is 1.62 cm^2/hr. /96

(d) The relative mean error in the calculation of a_S values, compared to the corresponding a_{Ts} values, is 22% for Koltushi, 25% for Kamennaya Step' and Pakhta-Aral, and 24% for all cases considered, with the root-mean-square deviation from this magnitude 16%.

3. Calculation of Fluxes of Heat Through the Soil and Daily Heat Cycle Sums

Eq. (17), taken from K. A. Sychev [7], was used to calculate the mean flux /97 of heat through the soil over certain time intervals, and the results obtained by using the formula derived from Eqs. (14'), (16), and (18) in G. Kh. Tseytin's paper [1] were used as "controls." The flux values obtained were equated on a conditional basis to the midpoints of the respective time intervals, followed by a comparison of the values obtained using the two methods indicated. The comparison showed that the individual flux values obtained by the K. A. Sychev method generally correctly reflect the characteristics of the daily course of the heat exchange in the soil [6]; that is, that there is a regular alternation in the daytime fluxes in the direction from the surface downward into the soil, followed by nighttime fluxes in the opposite direction. The change in the sign given the flux usually occurs during the transitional morning and evening hours. However, the discrepancy between individual flux values when equated to the same moment in time is quite substantial. Nor is there any pattern in the discrepancy between the calculated values during the transitional morning and evening hours, which are characterized by instability in directions and magnitudes of the fluxes of heat through the soil. The discrepancy does become quite obvious, however, during the day and night hours. Table 2 is provided by way of an example to show this. The mean value of the flux of heat through the soil obtained using K. A. Sychev's formula is designated Q_S, that using the G. Kh. Tseytin method, Q_{Ts}.

TABLE 1. Results of the Calculations Made for the Coefficient of Thermal Conductivity Using K. A. Sychev's and G. Kh. Tseytin's Formulas for Three Regions of the USSR on Individual Days in July, cm^2/hr.

Date	a_S	a_{Ts}	$a_S - a_{Ts}$	$\left(\frac{a_S - a_{Ts}}{a_{Ts}}\right)$, %
Koltushi, 1951 (time interval, 0800-1600)				
1	6.00	9.12	−3.12	34
4	10.00	5.51	4.49	81
7	3.62	4.15	−0.53	13
10	6.38	7.54	−1.16	15
13	5.63	5.67	−0.04	1
16	7.75	8.71	−0.96	11
19	−28.17	−43.50	—	—
22	7.88	9.40	−1.52	16
25	5.38	6.73	−1.35	20
28	3.75	4.70	−0.95	20
31	4.50	5.07	−0.57	11
			mean	22
Kamennaya Step', 1951 (time interval 0700-1900)				
3	4.10	3.59	0.51	14
4	2.34	3.34	−1.00	30
5	3.48	6.53	−3.05	47
7	10.40	7.02	3.38	48
8	5.51	9.30	−3.79	41
9	4.94	6.15	−1.21	20
10	5.47	6.44	−0.97	15
13	4.86	6.27	−1.41	22
14	5.00	6.80	−1.80	27
15	6.55	6.39	0.16	3
16	6.05	6.79	−0.74	11
21	9.40	12.95	−3.55	27
			mean	25
Pakhta-Aral, 1952 (time interval, 0830-2030)				
13	4.47	5.35	−0.88	16
15	3.75	5.75	−2.00	35
16	4.27	6.20	−1.93	31
18	4.70	6.07	−1.37	23
23	4.50	6.03	−1.53	25
25	4.84	6.31	−1.47	23
28	4.97	6.45	−1.48	23
			mean	25

TABLE 2. Fluxes of Heat Through the Soil, Calculated for Individual Nighttime and Daytime Hours Using the K. A. Sychev and G. Kh. Tseytin Formulas for the Three Regions of the USSR for Certain Days in July, cal/cm^2-min.

Date	Koltushi, 1951				Date	Kamennaya Step', 1951				Date	Pakhta-Aral, 1952	
	0000		1200			0100		1300			1230	
	Q_S	Q_{Ts}	Q_S	Q_{Ts}		Q_S	Q_{Ts}	Q_S	Q_{Ts}		Q_S	Q_{Ts}
1	−0,095	−0,038	0,032	0,030	3	−0,034	−0,026	0,038	0,008	—	—	—
2	−0,051	−0,034	0,118	0,048	4	−0,028	−0,012	—	—	9	0,212	0,150
3	−0,058	−0,028	0,042	0,017	7	−0,065	−0,032	0,080	0,048		—	—
4	−0,070	−0,042	0,057	0,010	8	−0,075	−0,035	0,104	0,062	13	0,196	0,130
5	−0,060	−0,024	0,032	0,042	9	−0,073	−0,032	0,123	0,076		—	—
6	−0,048	−0,026	—	—	10	−0,082	−0,034	0,119	0,058	15	0,170	0,130
12	−0,070	−0,049	0,158	0,099	13	−0,078	−0,031	0,103	0,054		—	—
13	−0,074	−0,040	0,142	0,076	14	−0,074	−0,031	0,116	0,062	16	0,170	0,130
14	−0,086	−0,020	0,105	0,062	15	−0,074	−0,038	0,098	0,047		—	—
15	−0,093	−0,050	0,146	0,074	16	−0,087	−0,028	0,103	0,054	18	0,195	0,110
16	−0,105	−0,049	0,114	0,060	17	−0,073	−0,029	—	—		—	—
17	−0,049	−0,032	0,112	0,012	—	—	—	—	—	23	0,191	0,150
18	−0,041	−0,034	—	—	—	—	—	—	—		—	—
26	−0,066	−0,037	0,107	0,062	—	—	—	—	—	25	0,218	0,150
27	−0,079	−0,047	0,136	0,084	—	—	—	—	—		—	—
28	−0,074	−0,034	0,094	0,059	—	—	—	—	—	28	0,188	0,110
29	−0,076	−0,036	0,111	0,060	—	—	—	—	—	—	—	—
30	−0,061	−0,027	0,019	0,018	—	—	—	—	—	—	—	—
31	−0,079	−0,040	—	—	—	—	—	—	—	—	—	—
No. cases	19	19	16	16	—	11	11	9	9	—	8	8
mean	−0,070	−0,036	0,095	0,051	—	−0,068	−0,030	0,098	0,052	—	0,192	0,132

Table 2 provides the following conclusions.

(a) The fluxes of heat calculated using K. A. Sychev's formula have higher absolute values as compared to those found using G. Kh. Tseytin's method, and this is true for both the daytime and nighttime hours.

(b) The absolute mean deviation for all Q_S values cited from the correspond- /98
ing Q_{Ts} values is 0.043 cal/cm^2-min when the root-mean-square error is 0.020 cal/cm^2-min, and is 112% for all cases.

(c) The mean relative deviation of the magnitudes of the flux of heat obtained using K. A. Sychev's formula from the magnitudes obtained by the G. Kh. Tseytin method is 102% at 0000 and 145% at 1200 for Koltushi, 127% at 0100 and 118% at 1300 hours for Kamennaya Step', and 47% at 1230 for Pakhta-Aral. The individual differences in values are even greater.

The goal of many calculations of fluxes of heat through the soil is to find the total heat collected, or given up, by the soil during some particular time interval, a 24-hour period, for example. So it is desirable to compare the daily totals of heat fluxes through the soil as calculated by the two methods indicated. To this end, both methods were used to calculate the mean flux of heat during several time intervals encompassing 24-hour periods. Successively overlapping ("sliding") intervals were used for the calculation involving K. A. Sychev's formula, but only adjacent intervals were used when the calculation was made using G. Kh. Tseytin's method. As before, the values obtained were equated to the midpoints of the corresponding intervals. And it was assumed, within the limits of from two to four hours during the day, and of up to six hours during the night, that the change in the time of the flux of heat through the soil takes place linearly in the first approximation. Using this as the base, the calculation of daily heat sums, in terms of flux values, equated to individual times of day, was made using the trapezoidal method. Table 3 lists the results. Here the daily, or daytime, heat sums calculated using the K. A. Sychev method are designated B_S, those using the G. Kh. Tseytin method B_{Ts}.

Table 3 shows the following features: There are, in Koltushi, positive (62% of the cases) and negative (32% of the cases) deviations in the daily B_S sums from the corresponding B_{Ts} sums. Yet, despite the fact that this was summer, the daily sums for individual days too were negative, and the only explanation would appear to be unstable weather. The excess of positive deviations over negative suggests that the excess of daytime Q_S over Q_{Ts} is, on the average, somewhat greater than the nighttime Q_S reduction as compared to Q_{Ts}. This is confirmed, in part, by the figures in Table 2, where it will be seen that in this region the excess of Q_S over Q_{Ts} at 1200 is, on the average, 0.044 cal/cm^2-min, whereas the mean nighttime value is 0.034 cal/cm^2-min. The rule in Kamennaya Step' (10 cases of 11, or 91%) is $B_S < B_{Ts}$, with two of the daily sums calculated using the K. A. Sychev method negative, something that is not entirely valid for this region and this season of the year, the more so because B_{Ts} is not negative in any of the 11 cases. The figures in Table 2 do not, at first glance, appear to confirm the result obtained, for here, as in Koltushi, the daytime excess of Q_S over Q_{Ts} is somewhat greater than the nighttime reduction. But it must be remembered that, first of all, the maximum values for the flux of heat through the soil takes place, on the average, not at 1300, but at an earlier hour

[6]. Second, when we calculate daily sums, the heat cycles can have a known value for the magnitude of heat fluxes not only during daytime and nighttime hours, as characterized by the figures in Table 2, but during the transitional time of the day as well. Finally, in all the cases for Pakhta-Aral, where the heat sum was calculated for daytime only, it was found that $B_S > B_{Ts}$, and this was to be expected because there always is a considerable excess of Q_S over Q_{Ts} during these hours in this region (see Table 2).

Table 3 provides the following conclusions.

(a) In the majority of cases (28 of 35, or 80%) the K. A. Sychev method correctly reflects the dynamics of the change in the daily sums for the heat cycle in the soil from day to day.

(b) There are significant discrepancies in absolute values, and sometimes /99 in signs as well, between individual values of daily sums for the heat cycles calculated for the same days using the two methods.

(c) The mean absolute deviation in the B_S sums from the corresponding B_{Ts} sums is 13.2 cal/cm^2 for all cases considered.

(d) The mean relative deviations in the sums for the heat cycle in the soil, calculated using the K. A. Sychev method, from the corresponding sums calculated using the G. Kh. Tseytin method, is 314% for Koltushi, 157% for Kamennaya Step', and 93% for Pakhta-Aral, with the figure 214% for all cases considered.

TABLE 3. Daily Sums for the Heat Cycle in the Soil, Calculated Using the K. A. Sychev and G. Kh. Tseytin Formulas for Three Regions of the USSR for Individual Days in July, cal/cm^2 per day.

Date	Koltushi, 1951			Date	Kamennaya Step', 1951			Date	Pakhta-Aral, 1952		
	B_S	B_{Ts}	$B_S - B_{Ts}$		B_S	B_{Ts}	$B_S - B_{Ts}$		B_S	B_{Ts}	$B_S - B_{Ts}$
1	−31.4	−11.0	−20.4	3	−5.0	0.5	−5.5	—	—	—	—
2	21.8	12.2	9.6	4	17.1	9.9	7.2	9	73.2	56.9	16.3
3	3.4	0.7	2.7	5	8.0	13.5	−5.5	—	—	—	—
4	−13.0	−0.5	−12.5	7	−6.2	11.0	−17.2	13	51.7	24.0	27.7
5	−4.3	6.2	−10.5	8	6.1	16.0	−9.9	—	—	—	—
12	34.3	20.4	13.9	9	11.6	23.4	−11.8	15	42.1	22.2	19.9
13	21.1	14.6	6.5	10	12.1	19.4	−7.3	—	—	—	—
14	7.3	−1.0	8.3	13	5.5	11.0	−5.5	16	45.5	21.0	24.5
15	55.5	10.6	44.9	14	9.7	14.6	−4.9	—	—	—	—
16	16.1	16.6	−0.5	15	6.5	17.5	−11.0	18	50.4	21.6	28.8
17	−3.4	−13.0	9.6	16	7.4	19.3	−11.9	—	—	—	—
26	6.5	5.8	0.7	—	—	—	—	23	46.1	28.8	17.3
27	33.1	22.1	11.0	—	—	—	—	—	—	—	—
28	−14.9	−7.0	−7.9	—	—	—	—	25	56.9	29.4	27.5
29	23.5	19.2	4.3	—	—	—	—	—	—	—	—
30	−29.0	−17.1	−11.9	—	—	—	—	28	51.0	25.8	25.2
No. cases	16	16	16	—	11	11	11	—	8	8	8
mean	7.9	4.9	10.9[2]	—	6.6	14.2	8.9[2]	—	52.1	28.7	23.4

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